

Initial and Final Conditions in Transient Analysis

Analysis of circuit behaviors at $t(0^-)$, $t(0^+)$, and $t(\infty)$ for series and parallel networks.

Definition of Initial & Final Conditions

Transient Period: The brief interval of time following a switching operation during which currents and voltages adjust from an old equilibrium state to a new state.

Initial Condition ($t = 0^+$): The state of the circuit immediately after a switching action occurs (at $t = 0$). It acts as the starting point for solving differential equations.

Pre-switching State ($t = 0^-$): The state of the circuit immediately before the switch changes position.

Final Condition ($t = \infty$): The steady-state condition reached by the circuit an infinitely long time after the switching operation, where all transient variations have completely decayed.

Virtues & Importance of Initial Conditions

Evaluation of Arbitrary Constants: Solving differential equations of networks yields general solutions with unknown constants. Initial values specify these constants uniquely.

Prevention of Physical Contradictions: Energy conservation dictates that certain states cannot change instantly. Managing initial states avoids mathematical anomalies.

Foundation for Laplace Transforms: Transforming differential equations into the s-domain inherently requires explicit initial values like $v(0+)$ and $i(0+)$.

Predictable Transient Trajectories: Knowing initial boundaries ensures engineers can predict maximum voltage stress and inrush currents on components safely.

Behavior of Circuit Elements

Resistor (R):

Governed by Ohm's Law: $v(t) = R * i(t)$. It has no memory and behaves identically at $t = 0+$, $t = 0-$, and $t = \infty$.

Inductor (L):

Current continuity rule: $i_L(0+) = i_L(0-)$. Energy cannot change instantly.

At $t = 0+$: If unenergized, it acts as an OPEN CIRCUIT to block immediate current changes.

At $t = \infty$: Acts as a SHORT CIRCUIT under DC steady state since $di/dt = 0$.

Capacitor (C):

Voltage continuity rule: $v_C(0+) = v_C(0-)$.

At $t = 0+$: If uncharged, it acts as a SHORT CIRCUIT to pull maximum initial inrush current.

At $t = \infty$: Acts as an OPEN CIRCUIT under DC steady state since $i = C * dv/dt = 0$.

Step-by-Step Evaluation Methodology

1. Analyze at $t = 0^-$: Assume the circuit has reached steady-state before the switch moves. Find currents through inductors $i_L(0^-)$ and voltages across capacitors $v_C(0^-)$.
2. Enforce Continuity Conditions : Set $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$. These are your core anchor values.
3. Draw Equivalent Circuit at $t = 0^+$: Replace inductors with current sources $[i_L(0^+)]$ or open circuits. Replace capacitors with voltage sources $[v_C(0^+)]$ or short circuits. Solve for unknown voltages/currents.
4. Differentiate Network Equations : To find derivatives like di/dt or dv/dt at $t = 0^+$, write KVL or KCL equations for $t > 0$, differentiate them with respect to t , and solve at $t = 0^+$.
5. Draw Equivalent Circuit at $t = \infty$: Replace inductors with short circuits and capacitors with open circuits. Solve for ultimate final values.

Evaluation in Series R-L-C Network

Configuration: DC Voltage source V applied to series R , L , C by closing a switch at $t = 0$. Initially unenergized.

At $t = 0^+$:

Since $i_L(0^-) = 0$ and $v_C(0^-) = 0$, we have $i(0^+) = 0$ and $v_C(0^+) = 0$.

The inductor acts as an open circuit, blocking current.

Finding $di(0+)/dt$:

Write KVL for $t > 0$: $R*i(t) + L*di(t)/dt + v_C(t) = V$

At $t = 0^+$: $R*(0) + L*di(0+)/dt + 0 = V \Rightarrow di(0+)/dt = V / L$

Finding $d^2i(0+)/dt^2$:

Differentiate KVL: $R*di/dt + L*d^2i/dt^2 + i/C = 0$

Substitute known values: $R*(V/L) + L*d^2i(0+)/dt^2 + 0 = 0 \Rightarrow d^2i(0+)/dt^2 = -R*V / L^2$

Evaluation in Parallel R-L-C Network

Configuration: Ideal step current source I applied to parallel R , L , C branches at $t = 0$.

At $t = 0+$:

If initially unenergized, $v_C(0+) = 0$ and $i_L(0+) = 0$.

Capacitor acts as a short circuit, pulling the entirety of source current: $i_C(0+) = I$.

Resistor and inductor currents are zero initially.

Finding $dv_C(0+)/dt$:

Write KCL: $v/R + i_L + C*dv_C/dt = I$

At $t = 0+$: $0 + 0 + C*dv_C(0+)/dt = I \Rightarrow dv_C(0+)/dt = I / C$

Final Steady State ($t = \infty$) :

Inductor acts as a short circuit, bypassing all other elements.

$v_C(\infty) = 0$, $i_L(\infty) = I$, $i_R(\infty) = 0$, $i_C(\infty) = 0$.